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The Impact of Discount Rate on Optimum Cut-off grade in Open-pit Mines

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ABSTRACT

Selecting an optimal cut-off grade is a pivotal concept in the design and planning of open-pit mine production, significantly impacting the economy of mining projects, particularly in metal mines. The mining process generally encompasses three main sectors: extraction, processing, and the final stages of smelting, refining, and marketing. The Lane model, along with the heuristic solution method introduced by Lane, underpins most marginal cut-off grade models. This paper presents a new model based on the Lane algorithm and the application of the discount rate. It incorporates the Net Present Value (NPV) of the mine plan and addresses the relevant extended nonlinear model to determine an optimal cut-off grade over the mine's lifespan. Fundamentally, NPV is affected by both discount rate and cut-off grade simultaneously. Consequently, this study examines the impact of varying discount rates on the marginal cut-off grade of a hypothetical copper mine using the developed model. The findings reveal that changes in the discount rate not only affect the cut-off grade but also the mine's lifespan and the capacities of the processing plant and sales. An increase in the discount rate elevates the cut-off grade, thereby shortening the mine's lifespan, whereas a decrease in the discount rate yields the opposite effect. The sensitivity analysis approach proposed in this paper aids mine designers in selecting the optimum cut-off grade.

KEYWORDS

Optimum cut-off grade, Optimization, Open-pit mines, Discount rate, Lane algorithm

I. INTRODUCTION

In mining, the cut-off grade is a crucial economic parameter that directly influences the economy, the ultimate pit limit, and its lifespan. This can alter the quantity of minable reserves and the mine's stripping ratio. The cut-off grade distinguishes the minerals in a deposit from waste materials. Waste can either be left in situ or transported to a disposal site, while minerals are processed at the processing plant and eventually sold (Ahmadi, 2018). Therefore, a meticulous analysis of the optimal methods for calculating the cut-off grade is essential, as employing an unsuitable cut-off grade can lead to financial losses. In practice, various cut-off grades are defined for different mining operations. These grades are directly dependent on market prices and costs, and indirectly on the grade distribution of the deposit (Githiria and Musingwini, 2018). Determining the optimal cut-off grade is a critical decision for mine engineers. Material with a mineral grade at or above the cut-off grade is classified as ore, while material below this grade is deemed waste (Ataei and Osanloo, 2003). Many factors that contribute to the determination of the cut-off grade. These factors can be classified into three groups: geological factors, economic factors, and operational factors (Saffariyan, Sayadi and Khodayari 2017). Bascetin (2007), Mohammed (1997), Ahmadi and Bazzazi (2020), Birch (2022), Biswas et al. (2020), Khan and Asad (2020), Paithankar et al. (2021) and Ataei and Osanloo (2003) have presented algorithms for determining the optimum cut-off grades for single or multiple metal deposits. For projects, the basic principle is that projects with a positive Net Present Value (NPV) should be undertaken. The main objective of each mining operation optimization is to maximize the NPV of the whole mining project (Rashidinezhad et al., 2008). Optimizing profit without accounting for the time value of capital can lead to disregarding mining capacity constraints, which ultimately lowering the net present value of production (Githiria and Musingwini, 2016). Over recent decades, the focus of cut-off grade optimization has been on maximizing NPV. This strategy necessitates the extraction of high-grade minerals during the early years of a mine's operation, followed by

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the extraction of progressively lower-grade materials. Essentially, the optimal cut-off grade is time-dependent, with a higher value in the initial years of the mine. The cut-off grade is a critical parameter that determines the volume of waste and mineral material, significantly impacting the cash flow of mining operations (Asad, 2016). Consequently, the operation's profitability and NPV are likely to increase. The cut-off grade also significantly influences the mine's lifespan and the overall profitability of mining operations. A higher cut-off grade increases the value per ton of material or ore, resulting in a shorter payback period and increased NPV. Thus, lower-risk investments allow investors, both governmental and private, to realize greater profits (Ahmadi, 2018). The maximization of NPV is a complex non-linear programming challenge. The Net Present Value (NPV) for a mining project hinges on a set of interrelated variables, including mining and milling capacities, the sequence of extraction, and the cut-off grade. Mine planning is the strategic process that establishes optimal values for these variables throughout the mine's operational life (Githiria and Musingwini, 2018). The primary aim of maximizing NPV in the initial years is to expedite the repayment of the capital invested in the project. This concept is known as the time value of money, which posits that the maximum economic return is realized when the future cash flows' highest NPV is generated (Githiria, 2016). A prevalent method for determining the optimal cut-off grade is the Lane algorithm which introduced by Lane in 1964. This algorithm has become foundational in the field, particularly through his seminal paper, "Selecting the Optimal Cut-off Grade (Lane, 1964). It addresses two scenarios: profit maximization and NPV optimization. Lane's algorithm presumes fixed capacities across mining, processing, smelting, and refining operations (Hustrulid and Kuchta, 1995). Lane concluded that in addition to economic parameters, the capacity and distribution of reserve grade-tonnage are crucial in calculating the cut-off grade, selecting it according to the available blocks, referred to as optimum cut-off grade or the dynamic one. Mine production planning and management of open-pit mines, especially for mines with a long life, is a complex process. One of the initial critical decisions following the design of the ultimate pit limit is the production planning and determination of the cut-off grade, which serves as a nexus between the project's technical and economic parameters. This decision is pivotal in aligning the mine's operational strategy with its financial objectives (Ahmadi, 2018).

Fundamentally, the discount rate influences the NPV, which is also affected by the cut-off grade. In this paper, a new non-linear model based on Lane's model will be developed. Also, Simulated Annealing (SA) solver will be used to solve the presented model. The extended model will be a two-component model that will optimized the NPV of mine plan by considering the simultaneous role of the discount rate and the cut-off grade. Also, the sensitivity analysis approach will be proposed in this paper for mine designers in selecting the optimum cut-off grade.

II. Materials and Methods

A. *Cut-off grade Optimization*

The selling price of metal and the distribution of grade tonnage are critical input parameters for developing a cut-off grade policy. When determining the cut-off grade for the break-even stripping ratio at various stages such as mining capacity, concentration plant capacity, and refining plant capacity the grade distribution and the time value of money are often overlooked (Asad et al., 2016). Therefore, this grade cannot be the optimal cut-off grade. One of the most important parameters in designing an open pit mine is determining the optimum grade. The most typical indicators used in optimization are: Maximum net profit, maximum metal quantity and optimum mine life. The production of metal (copper) from the mine to the market includes three common extraction stages, processing, and refining, which are schematically shown in Fig. 1.

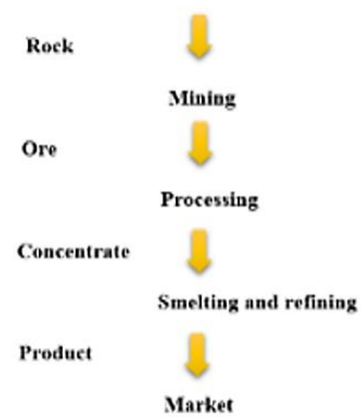

Fig. 1. Steps of copper production operation from mine to market

Eq.(1) shows the calculation of profit parameters by using the Lane algorithm:

$$P = (S - r) \times Q_r - [m \times Q_m - c \times Q_c - f \times T] \qquad (1)$$

The symbols used in this relation are introduced in Table 1, among these items, the mining capacity (Q_m) is independent of the cut-off grade, the processing capacity (Q_c) is dependent on the cut-off grade (with the increase of the cut-off grade, the processing capacity decreases). As a result, the sales capacity (Q_r) will be a function of the cut-off grade (a strictly downward function) and also the life of the mine (T) will be a function of the cut-off grade, because the mine lifespan will change by changes in the plant capacity. According to Lane's theory, the profit function can be investigated in six states, the details are available (Hustrulid and Kuchta, 1995) . In Table 1, Some other symbols are shown, which can be used in other stages. The characteristics of the materials in the hypothetical mine cavity under study are shown in Table 2 and the grade and tonnage information of the mine are shown in Table3 (Lane, 1964). The information related to the economic parameters and the capacity of different stages of mining are also shown in Table 4.

Table 1. Parameters of the objective function relationship (profit)

Related symbol	Parameter
Q_m	Mining tonnage
Q_c	Processing plant tonnage
Q_r	Smelting and refining tonnage(sales)
m	Mining cost
c	Milling cost
r	Smelting and refining cost
f	Fixed cost
T	Mine life
M	Mining capacity
C	Processing plant capacity
R	Smelting and refining capacity
S	Sale price
P	Profit
Y	Recovery
i	Discount Rate
CF	Total sum of cash flow during mine life

In this paper, based on the profit function of the Lane model, applying the discount rate and taking into account the existing limitations, a new model has been developed which, taking into account the net present value of the plan, chooses an optimal cut-off grade for the mine's life years. The discount rate directly affects the amount of net present value, and the net present value is also affected by the cut-off grade. The developed model is implemented on the data of a hypothetical copper mine.

B. Define Model Parameters and Decision Variables

The goal of the model is to maximize the net present value that includes the time value of money in the calculations. The decision variables in this model are cut-off grade (g), average grade ($\bar{g}$), mine life (T), processing plant capacity (Q_c) and smelting and refining (Q_r). If the relationship between the capacity of the processing plant and the sales capacity, as well as the average rate (Q_c, Q_r) is written in terms of the cut-off grade (using the information in Table 5 and establishing a proper correlation between the cut-off grade and the capacity of the processing plant), a non-linear equation will exist. The variation of ore tonnage and average grade toward cut-off grade are illustrated in Fig.s 1 and 2. Additionally, the coefficient values have been computed in Eq.s 2 and 3.

Table 2. Characteristics of the material inside the final pit

Cut-off grade (%)	Average grade (%)	Rock tonnage (10^3tonnes)
0-0.1	0.05	395.32
0.1-0.2	0.17	37.96
0.2-0.3	0.24	57.6
0.3-0.4	0.33	77.24
0.4-0.5	0.44	96.88
0.5-0.6	0.58	116.52
0.6-0.7	0.64	98.89
0.7-0.8	0.75	51.39
0.8-0.9	0.85	28.35
0.9-1	0.95	14.98
1-1.1	1.05	7.39
1.1-1.2	1.15	5.23
1.2-1.3	1.25	3.51
1.3-1.4	1.35	2.93
1.4-1.5	>1.4	5.81

Table 3. Grade-Tonnage

Cut-off grade (%)	Cumulative average grade (%)	Ore tonnage 10^3 (tonnage)	Metal (ton)
0	0.35	1000	2975
0.1	0.54	604.68	2775
0.2	0.57	566.72	2745
0.3	0.61	509.12	2640
0.4	0.66	431.88	2423
0.5	0.72	335	2050
0.6	0.79	218.48	1467
0.7	0.92	119.59	935
0.8	1.05	68.2	608.7
0.9	1.19	39.85	403
1	1.33	24.87	281
1.1	1.45	17.78	215
1.2	1.57	12.25	164
1.3	1.70	8.74	127
>1.4	1.88	5.81	93

Table 4. Economic parameters of the hypothetical mine

Parameters	Units	values
Mining cost	$/ton Ore	3
Processing cost	$/ton Concentrate	4
Smelt, Refine & Sell cost	$/ton Metal	1500
Price	$/ton Metal	7000
Fixed cost	$/year	7000
Recovery	%	85
Mining capacity	1000 t/year	50
Plant capacity	1000 t/year	15
Market(or smelting) capacity	t/year	100

$$Q_c = -982g^2 - 85g + 623,\quad R^2{=}0.97 \quad (2)$$
$$\bar{g} = 0.61g^2 + 0.07g + 0.53, R^2{=}0.97 \quad (3)$$
$$Q_r = \bar{g}Q_c y \quad (4)$$
$$Q_r = 0.85(-982g^2 - 85g + 623)(0.61g^2 + 0.07g + 0.53) \quad (5)$$
$$Max\ NPV = \sum_{j=1}^{T} \frac{C_F}{T} \times \frac{1}{i(1+i)^j} \quad (6)$$

s.t:

$$T \geq 20$$
$$T \geq \frac{Q_c}{15000}$$
$$T \geq \frac{Q_r}{100}$$
$$Q_c - 982g^2 - 85g + 623 = 0$$
$$\bar{g} - 0.61g^2 + 0.07g + 53 = 0$$
$$Q_r - 0.85\bar{g}Q_c = 0$$
$$C_F = (7000 - 1500)Q_r - 3Q_m - 4Q_c - 7000T$$
$$g, \bar{g}, Q_c, Q_r \geq 0$$

III. DISCUSSION AND RESULTS

A. *Model Solution Results for a Hypothetical Mine*

Considering Eq.s (1) to (5), the objective function and limitations of the model are defined for the existing hypothetical mine using the symbols and parameters of Tables 1-4.

With the basis of Eq. (6), which is specific to the hypothetical mine, and using Mathematica software, calculations were made for different discount rates and reported in Table 5. The results of solving the model include the determination of the optimal grade, the average grade of the factory, the mine's life, the net present value of the mine, the cash flow of the mine, the constant annual cash flow, the amount of ore that can be processed and finally the amount of pure metal produced, which are reported in Table 5 with different discount rates.

As indicated in Table 5, an increase in the discount rate results in a higher cut-off grade. This escalation in the cut-off grade leads to a decrease in the average grade, as well as the capacities of the processing plant and the smelting, refining, and sales operations. Consequently, these changes contribute to a shortened mine life. However, there is a decrease in both the discount rate and the cut-off grade. As a result, the average grade of the plant's feed input and the processing plant's capacity experience an increase over the mine's lifespan. Similarly, the capacities for smelting, refining, and sales, along with the mine life, also see an uptick.

Fig. 2 illustrates that the cut-off grade's upward trajectory intensifies as the discount rate climbs from zero to 15 percent. The rate of change is steeper between zero to 5 percent, after which the intensity diminishes, leading to a more gradual slope in the discount rate curve. By interpreting these trends and considering the inflation and interest rates announced by the central banks of each country, it becomes feasible to ascertain the optimal cut-off grade that aligns with the economic conditions of each country.

In Fig. 3, a bar chart illustrating the variations in the mine life with different discount rates is depicted. The bar chart shows the changes in the processing plant's capacity with different discount rates, shown in Fig. 4. A bar graph in Fig. 5 shows the changes in the final saleable product of the mine concerning various discount rates. The visible decrease in the production of the final product with increasing discount rates is evident.

In Fig. 6, the bar graph of changes in the net present value of the mine is determined for different discount rates. Fig. 7 also shows the 3D graph in which the value of the net present value of the plan is determined for the life and different cut-off grades.

Table 5. Calculation results for different discount rates in Mathematica (https://www.wolfram.com/mathematica/)

i (%)	g_c (%)	$\bar{g}$ (%)	T (year)	NPV (10^3\$)	CF (10^3\$)	CF/T	Q_c (10^3tons)	Q_r (tons)
0	0.05	0.53	41.06	9662.25	9662.43	235.3264	616.295	2802
1	0.14	0.54	39.32	7919.3	9623.95	244.7597	591.853	2775
2	0.22	0.57	37	6645.5	9478.64	256.1795	556.771	2720
5	0.35	0.62	31.47	4383.35	8800.46	279.646	472.955	2529
6	0.37	0.63	30.1	3917.4	8618.07	286.3146	457.114	2484
8	0.41	0.66	28.04	3215.14	8189.97	292.0817	423.076	2377
10	0.43	0.67	26.48	2713.67	7952.61	300.3252	404.878	2315
12	0.45	0.68	25.3	2339.48	7637.58	301.8806	385.895	2246

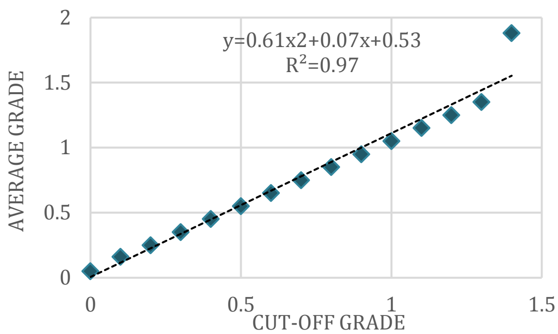

Fig. 2. Average grade variation versus cut-off grade

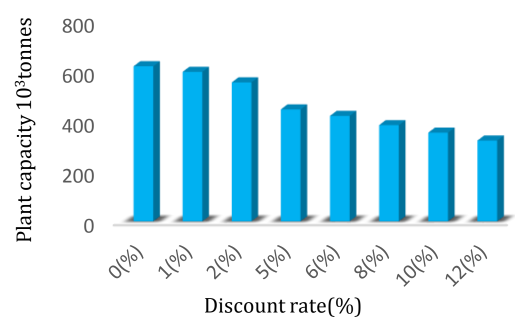

Fig. 6. Plant capacity variation of discount rate

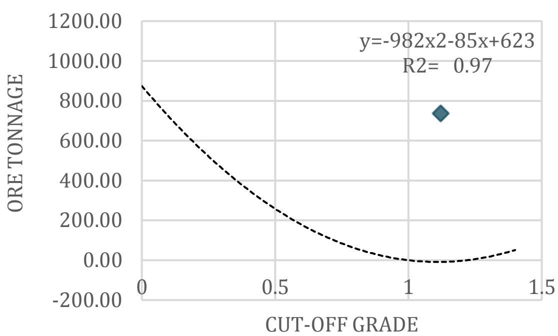

Fig. 3. Ore tonnage variation versus cut-off grade

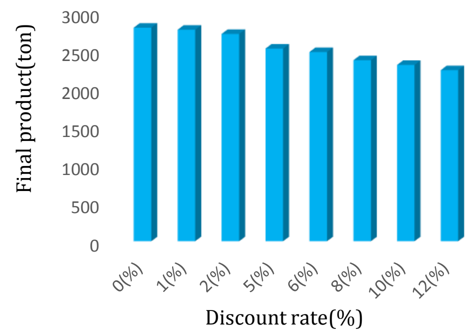

Fig. 7. Final product variation versus discount rate

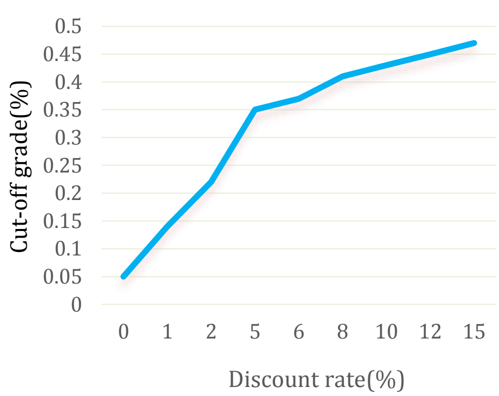

Fig. 4. Cut-off grade variation versus discount rate

Fig. 8. Net present value variation versus discount rate

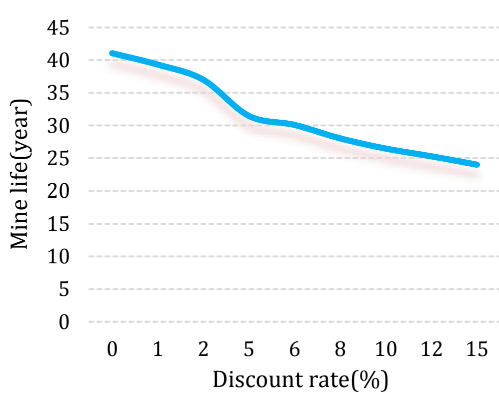

Fig. 5. Mine life variation versus discount rate

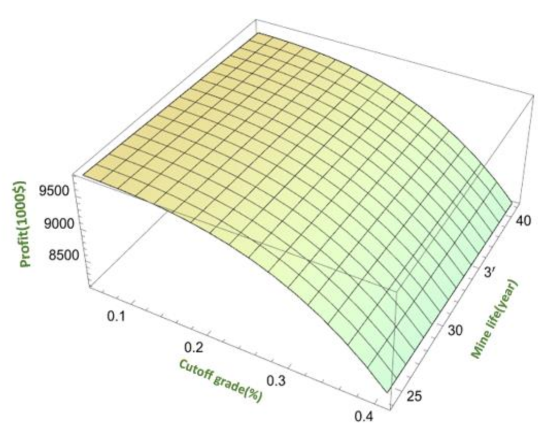

Fig. 9. The 3D graph of the Net Present Value of the plan for different life and cut-off grade

IV. CONCLUSIONS

The cut-off grade is a critical parameter in the design of open-pit mines. Variations in the cut-off grade affect the average grade and the processing plant's capacity for processing, smelting, and refining. Recently, the focus has been on developing models and equations that lead to calculating the optimal cut-off grade, thereby maximizing the Net Present Value (NPV). Applying appropriate discount rates by each country's policy is essential. Considering the discount rates announced by central banks and the interest rates of bonds can assist mine engineers in selecting the most suitable option. Consequently, mine designers and investors must examine the annual time series data related to the discount rate. In this study, a sensitivity analysis was performed to illustrate the impact of discount rate fluctuations on the NPV maximization model, to identify the optimal cut-off grade for discount rates ranging from 0% to 15% for a hypothetical copper mine. Based on the output of the used model, an increase in the discount rate has led to a rise in the cut-off grade. This, in turn, causes a decrease in the average grade and the capacities of the processing plant, as well as smelting, refining, and sales operations. Consequently, these changes result in a shortened mine life. Conversely, a decrease in the discount rate leads to a lower cut-off grade, which enhances the average feed grade of the plant throughout the mine's life and boosts the processing plant's capacity. Following the explanations provided, the capacities for smelting, refining, and sales, along with the mine's lifespan, are also expected to increase. By considering the sensitivity analysis approach outlined in this paper, alongside the economic conditions of each country and the technical specifics of the mine, engineers can determine the optimal cut-off grade for the plant.

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